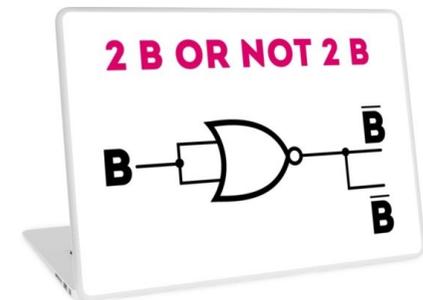
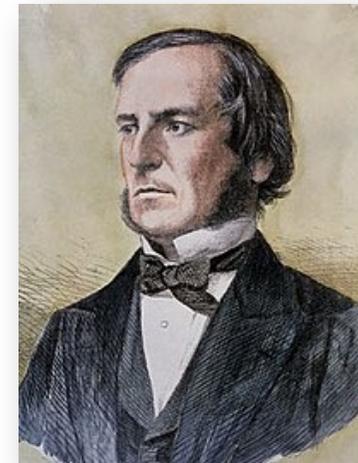




UNIVERSITÀ  
DI PARMA



# algebra di Boole



## algebra di Boole

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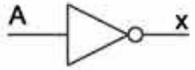
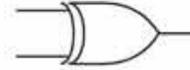
- l'algebra di Boole è un **formalismo** che opera su variabili (*variabili booleane*)
- le **variabili booleane** possono assumere due soli valori: **vero**, **falso**
- sulle variabili booleane è possibile definire un insieme di funzioni (*funzioni booleane*)
- il risultato di una **funzione booleana** può assumere solo il valore **vero** o il valore **falso**
- il valore **vero** viene anche rappresentato con **1** e il valore **falso** con **0**

## funzioni e tabella di verità

- una **tabella di verità** *definisce* una funzione booleana
  - stabilisce il valore risultante per ciascuna **combinazione** dei valori in ingresso
- a volte, *specifica incompleta*  
(*certe combinazioni di ingressi non possono verificarsi*) → *non è specificato alcun valore*

#	w	x	y	z	f
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

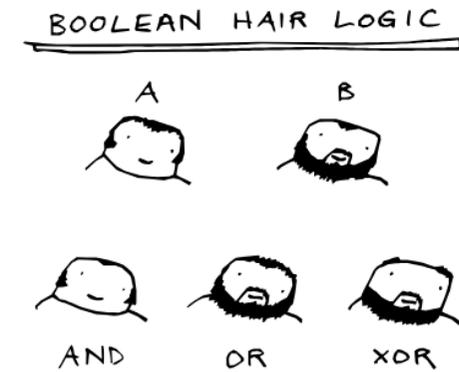
## operatori di base

Name	NOT	AND	NAND	OR	NOR	XOR	XNOR																																																																																																
Alg. Expr.	$\bar{A}$	$AB$	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A \oplus B$	$\overline{A \oplus B}$																																																																																																
Symbol																																																																																																							
Truth Table	<table border="1"> <thead> <tr> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> </tr> </tbody> </table>	A	X	0	1	1	0	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	B	A	X	0	0	0	0	1	0	1	0	0	1	1	1	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	B	A	X	0	0	1	0	1	1	1	0	1	1	1	0	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	B	A	X	0	0	0	0	1	1	1	0	1	1	1	1	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	B	A	X	0	0	1	0	1	0	1	0	0	1	1	0	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>0</td> </tr> </tbody> </table>	B	A	X	0	0	0	0	1	1	1	0	1	1	1	0	<table border="1"> <thead> <tr> <th>B</th> <th>A</th> <th>X</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table>	B	A	X	0	0	1	0	1	0	1	0	0	1	1	1
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## espressione booleana

- operatori possono essere **combinati** in espressioni
  - altra forma di definizione di funzioni booleane
  - es.  $F_2(A, B, C) = A \cdot B + C$

Operatore	Simbolo
And	$\cdot (\wedge)$
Or	$+ (\vee)$
Not	$\neg$
Xor	$\oplus$
Nand	$\uparrow$
Nor	$\downarrow$



## proprietà degli operatori

Proprietà	Not
Complemento	$\neg\neg A = A$

Proprietà	And	Or
Commutativa	$A \cdot B = B \cdot A$	$A + B = B + A$
Associativa	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	$(A + B) + C = A + (B + C)$
Distributiva	$A + (B \cdot C) = (A + B) \cdot (A + C)$	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
Idempotenza	$A \cdot A = A$	$A + A = A$
Identità	$A \cdot 1 = A$	$A + 0 = A$
Del limite	$A \cdot 0 = 0$	$A + 1 = 1$
Assorbimento	$A \cdot (A + B) = A$	$A + (A \cdot B) = A$
Inverso	$A \cdot \neg A = 0$	$A + \neg A = 1$
De Morgan	$\neg(A \cdot B \cdot C \dots) = \neg A + \neg B + \neg C \dots$	$\neg(A + B + C \dots) = \neg A \cdot \neg B \cdot \neg C \dots$

*Attenzione a De Morgan: errore comune!*

## leggi di De Morgan

*le leggi di De Morgan sono principi della logica proposizionale che stabiliscono come negare le congiunzioni e le disgiunzioni*

A	B	A+B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

A	B	A.B	$\overline{A.B}$	$\overline{A}$	$\overline{B}$	$\overline{A+B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

## forme canoniche

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- le **forme canoniche** sono espressioni standardizzate di una funzione booleana e permettono di rappresentarla in modo univoco
- **somma di prodotti (SP)**: si considerano le righe a **1**
  - $F1(A, B, C) = (\neg A \neg B \neg C) + (\neg A \cdot B \cdot C) + (A \neg B \cdot C) + (A \cdot B \neg C) + (A \cdot B \cdot C)$
- **prodotto di somme (PS)**: si considerano le righe a **0**
  - $F1(A, B, C) = (A + B + \neg C) \cdot (A + \neg B + C) \cdot (\neg A + B + C)$

## somma di prodotti

<i>A</i>	<i>B</i>	<i>C</i>	<i>F</i>	
0	0	0	1	→ <i>SP</i>
0	0	1	0	
0	1	0	0	
0	1	1	1	→ <i>SP</i>
1	0	0	0	
1	0	1	1	→ <i>SP</i>
1	1	0	1	→ <i>SP</i>
1	1	1	1	→ <i>SP</i>

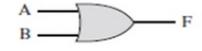
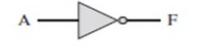
$$F(A, B, C) := (\neg A \cdot \neg B \cdot \neg C) + (\neg A \cdot B \cdot C) + (A \cdot \neg B \cdot C) + (A \cdot B \cdot \neg C) + (A \cdot B \cdot C)$$

## prodotto di somme

<i>A</i>	<i>B</i>	<i>C</i>	<i>F</i>	$\neg F$	
0	0	0	1	0	
0	0	1	0	1	$\rightarrow PS$
0	1	0	0	1	$\rightarrow PS$
0	1	1	1	0	
1	0	0	0	1	$\rightarrow PS$
1	0	1	1	0	
1	1	0	1	0	
1	1	1	1	0	

$$F(A, B, C) := (A + B + \neg C) \cdot (A + \neg B + C) \cdot (\neg A + B + C)$$



Name	Graphical Symbol	Algebraic Function	Truth Table															
AND		$F = A \cdot B$ or $F = AB$	<table border="1"><thead><tr><th>A</th><th>B</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></tbody></table>	A	B	F	0	0	0	0	1	0	1	0	0	1	1	1
A	B	F																
0	0	0																
0	1	0																
1	0	0																
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OR		$F = A + B$	<table border="1"><thead><tr><th>A</th><th>B</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></tbody></table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	1
A	B	F																
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1	0	1																
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NOT		$F = \bar{A}$ or $F = A'$	<table border="1"><thead><tr><th>A</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></tbody></table>	A	F	0	1	1	0									
A	F																	
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NAND		$F = \overline{AB}$	<table border="1"><thead><tr><th>A</th><th>B</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></tbody></table>	A	B	F	0	0	1	0	1	1	1	0	1	1	1	0
A	B	F																
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1	0	1																
1	1	0																
NOR		$F = \overline{A + B}$	<table border="1"><thead><tr><th>A</th><th>B</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></tbody></table>	A	B	F	0	0	1	0	1	0	1	0	0	1	1	0
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XOR		$F = A \oplus B$	<table border="1"><thead><tr><th>A</th><th>B</th><th>F</th></tr></thead><tbody><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></tbody></table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	0
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